

Arithmetic/ Geometric Sequences and

Series

Name: Key

1. Identify which of the following sequences are arithmetic. For each arithmetic sequence, state the values of t_1 and d , and the next three terms.

a) 4, 7, 10, 13, ... $d=3$
 $t_1=4$

b) 5, 15, 45, 135, ...
 $\times 3 \times 3$

Geometric

c) $x, x+2, x+4, x+6, \dots$
 $+2 \quad +2 \quad +2$
 $d=2$
 $t_1=x$
 $x+9, x+12, x+15$

2. Write the first four terms of each arithmetic sequence for the given values of t_1 and d .

a) $t_1 = -5, d = -2$

$-5, -7, -9, -11$

b) $t_1 = 3, d = x$

$3, 3+x, 3+2x, 3+3x$

3. Given the general term, state the first four terms of each sequence.

a) $t_n = 13 - 3n$
 $13 - 3(1) = 10$
 $13 - 3(2) = 7$

$10, 7, 4, 1,$

b) $t_n = \frac{1}{2}n + 4$

$\frac{1}{2}(1) + 4 = \frac{9}{2}$ $\frac{9}{2}, \frac{11}{2}, 6$
 $\frac{1}{2}(2) + 4 = 5$

4. Determine the general term and the 50th term for each arithmetic sequence.

a) 6, 10, 14, ...
 $t_n = 6 + (n-1)4$
 $t_n = 6 + 4n - 4$
 $t_n = 2 + 4n$
 $t_{50} = 2 + 4(50) = 202$

b) $3, 2\frac{1}{2}, 2, \dots$
 $t_n = 3 + (n-1)(-\frac{1}{2})$
 $t_{50} = \frac{7}{2} - \frac{50}{2}$
 $= -\frac{43}{2}$ or -21.5
 $t_n = \frac{7-n}{2}$

5. Determine the number of terms in each finite arithmetic sequence.

a) $-6, -3, 0, \dots, 222$

$222 = -6 + (n-1)3$
 $\frac{228}{3} = \frac{3(n-1)}{3}$

$76 = n-1$ $n = 77$

b) $3\frac{1}{4}, 3\frac{3}{4}, 4\frac{1}{4}, \dots, 15\frac{3}{4}$

$+ \frac{2}{4}$ $15\frac{3}{4} = 3\frac{1}{4} + (n-1)(\frac{2}{4})$

$\frac{63}{4} = \frac{13}{4} + \frac{2n}{4} - \frac{2}{4}$

$63 = 13 + 2n - 2$

6. Determine the unknown terms in each arithmetic sequence.

a) 4, \square , \square , 16
 $16 = 4 + (4-1)d$
 $16 = 4 + 3d$
 $12 = 3d$
 $\frac{12}{3} = \frac{3d}{3}$

b) 20, \square , \square , \square , \square , \square , -10
 $-10 = 20 + (6-1)d$
 $\frac{-30}{5} = \frac{15d}{5}$
 $d = -6$
 $52 = \frac{2n}{2}$
 $n = 26$

7. The 20th term of an arithmetic sequence is 107, and the common difference is 5.

Determine the first term, the general term, and the 40th term of this sequence.

$$t_{20} = 107 \quad d = 5$$

$$107 = t_1 + (20-1)5$$

$$107 = t_1 + 14 \cdot 5$$

$$107 = t_1 + 95$$

$$t_1 = 12$$

$$t_n = 12 + (n-1)5$$

$$t_n = 12 + 5n - 5$$

$$t_n = 7 + 5n$$

$$t_{40} = 7 + 5(40)$$

$$= 7 + 200$$

$$= 207$$

8. Use the two given terms to find t_1 , d , and t_n for each arithmetic sequence.

a) $t_{11} = 25, t_{30} = 101$

$$101 = t_1 + 29d$$

$$25 = t_1 + 10d$$

$$101 - 25 = t_1 + 29d - t_1 - 10d$$

$$76 = 19d$$

$$d = 4$$

$$25 = t_1 + 10(4)$$

$$25 = t_1 + 40$$

$$t_1 = -15$$

$$t_n = -15 + (n-1)4$$

$$t_n = -15 + 4n - 4$$

$$t_n = -19 + 4n$$

b) $t_2 = 90, t_{51} = -57$

$$-57 = t_1 + (50)d$$

$$90 = t_1 + 1d$$

$$-57 - 90 = t_1 + 50d - t_1 - 1d$$

$$-147 = 49d$$

$$d = -3$$

$$90 = t_1 + 1(-3)$$

$$90 = t_1 - 3$$

$$t_1 = 93$$

$$t_n = 93 + (n-1)(-3)$$

$$t_n = 93 - 3n + 3$$

$$t_n = 96 - 3n$$

9. The terms $5 + x$, 8 , and $1 + 2x$ are consecutive terms in an arithmetic sequence.

Determine the value of x and state the three terms.

$$8 - (5+x) = d$$

$$1 + 2x - 8 = d$$

$$8 - (5+x) = 1 + 2x - 8$$

$$8 - 5 - x = -7 + 2x$$

$$3 - x = -7 + 2x$$

$$10 = 3x$$

$$x = \frac{10}{3}$$

$$5 + \frac{10}{3}$$

$$\frac{15}{3} + \frac{10}{3} = \frac{25}{3}$$

$$1 + 2\left(\frac{10}{3}\right) = \frac{20}{3} + \frac{20}{3} = \frac{40}{3}$$

$$\frac{25}{3}, 8, \frac{23}{3}$$

$$\frac{25}{3}, \frac{24}{3}, \frac{23}{3}$$

10. The triangular shapes are made from asterisks.



Figure 1

Figure 2

Figure 3

a) How many asterisks will be in the fourth triangle? the fifth triangle?

b)

$$t_4 = 15 \quad t_5 = 18$$

b) Write the general term for the sequence involving the number of asterisks in the triangles.

$$t_n = 6 + (n-1)3$$

$$t_n = 6 + 3n - 3$$

$$t_n = 3 + 3n$$

c) How many asterisks will be in the 20th diagram?

$$t_{20} = 3 + 3(20) \Rightarrow t_{20} = 63$$

d) Which diagram will contain 126 asterisks?

$$t_{126} = 3 + 3(126) \Rightarrow t_{126} = 381$$