Arithmetic/ Geometric Sequences and Series

Name: Key

1. Determine the sum of each arithmetic series.

a) $14 + 10 + 6 + \cdots + (-86)$

$t_1 = 14$
$d = -4$
$n = ?$
$t_n = -86$

$-86 = 14 + (n-1)(-4)$
$-100 = -4n + 4$
$-104 = -4n$
$n = 26$
$S_n = \frac{26}{2}(14 - 86)$
$S_n = 13(-72) = -936$

b) $\frac{3}{4} + \frac{8}{4} + \frac{13}{4} + \cdots + \frac{49}{2}$

$t_1 = \frac{3}{4}$
$d = \frac{5}{4}$
$n = ?$
$t_n = \frac{49}{2}$

$\frac{49}{2} = \frac{3}{4} + (n-1)(\frac{5}{4})$
$\frac{49}{2} - \frac{3}{4} = (n-1)\frac{5}{4}$
$\frac{95}{4} = \frac{5}{4}n - \frac{5}{4}$
$95 = 5n - 5$
$\frac{100}{5} = \frac{5n}{5}$
$n = 20$
$S_n = \frac{20}{2}(\frac{3}{4} + \frac{49}{2})$
$S_n = 10(\frac{101}{2}) = 505$

2. For each arithmetic series, determine the indicated sum.

a) $4 + 9 + 14 + \ldots$; first 12 terms

$S_n = \frac{n}{2}(2t_1 + (n-1)d)$
$= \frac{12}{2}(2(4) + (12-1)5)$
$= 6(8 + 55)$
$= 6(63) = 378$

b) $x + 3x + 5x + \ldots$; first 20 terms

$S_n = \frac{20}{2}(2(x) + (20-1)2x)$
$= 10(2x + (19)2x)$
$= 10(2x + 38x)$
$= 10(40x)$
$= 400x$

3. For each arithmetic series, determine the number of terms.

a) $t_1 = 3,\ t_n = 59,\ S_n = 465$

$n = ?$
$S_n = \frac{n}{2}(t_1 + t_n)$
$465 = \frac{n}{2}(3 + 59)$
$465 = \frac{n}{2}(62)$
$\frac{465}{31} = \frac{31n}{31}$
$n = 15$

b) $t_1 = -2,\ t_n = -74,\ S_n = -950$

$S_n = \frac{n}{2}(t_1 + t_n)$
$-950 = \frac{n}{2}(-2 - 74)$
$-950 = \frac{n}{2}(-76)$
$\frac{-950}{-38} = \frac{-38n}{-38}$
$n = 25$

4. For each arithmetic series, determine the 12th term and the 12th partial sum.

a) $3 - 1 - 5 - \ldots$

$t_n = 3$
$d = -4$
$n = 12$

$t_{12} = 3 + (12-1)(-4)$
$t_{12} = 3 + (11)(-4)$
$t_{12} = -41$
$S_{12} = \frac{12}{2}(3 - 41)$
$S_{12} = 6(-38) = -228$

b) $\frac{3}{5} + \frac{7}{5} + \frac{11}{5} + \ldots$

$t_{12} = \frac{3}{5} + (12-1)(\frac{4}{5})$
$t_{12} = \frac{3}{5} + (11)\frac{4}{5}$
$t_{12} = \frac{3}{5} + \frac{44}{5}$
$t_{12} = \frac{47}{5}$
$S_{12} = \frac{12}{2}(\frac{3}{5} + \frac{47}{5})$
$= 6(\frac{3}{5} + \frac{47}{5})$
$= 6(\frac{50}{5})$
$= 6(10) = 60$

5. Determine the sum of each arithmetic series, given the first and nth terms.

a) $t_1 = -3,\ t_{14} = 62$

$S_n = \frac{14}{2}(-3 + 62)$
$= 7(59)$
$= 413$

b) $t_1 = \sqrt{3},\ t_{10} = 18\sqrt{3}$

$S_n = \frac{10}{2}(\sqrt{3} + 18\sqrt{3})$
$= 5(19\sqrt{3})$
$= 95\sqrt{3}$

6. Determine the sum of all multiples of 7 between 1 and 1000.

$\frac{1000}{7} = 142.85$

$142 \times 7 = 994$

$994 = 7 + (n-1)7$ $\quad n = 142$

$-7 \quad -7$

$987 = 7n - 7$

$\frac{994}{7} = \frac{7n}{7}$

$S_n = \frac{142}{2}(7 + 994)$

$= 71(1001)$

$= 71071$

7. In an arithmetic series, the third term is 24 and the sixth term is 51. What is the sum of the first 25 terms of the series?

$t_3 = 24$

$t_6 = 51$

$d = \frac{51 - 24}{6 - 3}$

$\frac{27}{3} = 9$

$51 = t_1 + (6-1)(9)$ $\quad t_1 = 6$

$51 = t_1 + 5(9)$

$51 = t_1 + 45$

$-45 \quad -45$

$S_n = \frac{n}{2}(2t_1 + (n-1)d)$

$= \frac{25}{2}(2(6) + (25-1)(9))$

$= \frac{25}{2}(12 + 24(9))$

$= \frac{25}{2}(12 + 216)$

$= \frac{25}{2}(228)$

$= 25(114)$

$= 2850$

8. The sum of the first eight terms of an arithmetic series is 176. The sum of the first nine terms is 216. Determine the first and ninth terms of the series.

$S_8 = 176$

$S_9 = 216$

$S_9 - S_8 = d$

$216 - 176 = d$

$d = 40$

$S_n = \frac{n}{2}(2t_1 + (n-1)d)$

$216 = \frac{9}{2}(2t_1 + (9-1)40)$

$216 = \frac{9}{2}(2t_1 + 8(40))$

$432 = 9(2t_1 + 320)$

$432 = 18t_1 + 2880$

$-2880 \quad -2880$

$-2448 = 18t_1$

$-136 = t_1$

$t_9 = -136 + (9-1)(40)$

$= -136 + 8(40)$

$= -136 + 320$

$= 184$

9. The sum of the first n terms of an arithmetic series is $S_n = 3n^2 + 4n$.

a) Determine the first five partial sums.

$S_n = 3(5)^2 + 4(5)$

$= 3(25) + 20$

$= 75 + 20$

$= 95$

b) Determine the first five terms of the series.

$S_1 = 3(1)^2 + 4(1)$

$= 3 + 4$

$= 7$

$S_2 = 3(2)^2 + 4(2)$

$= 12 + 8$

$= 20$

$t_1 = 7$

20

7, 13, 19, 25, 31

c) Use the formula to verify that the sum of the first five terms is equal to S_5.

$7 + 13 + 19 + 25 + 31 = 95$

10. A student is offered the opportunity to earn \$6.00 for the first day, \$11.00 for the second day, \$16.00 for the third day, and so on, for 20 working days. Or, the student can accept \$1000 for the whole job. Which offer pays more?

$t_1 = 6$

$d = 5$

$n = 20$

$S_n = \frac{20}{2}(2(6) + (20-1)5)$

$= 10(12 + 19(5))$

$= 10(12 + 95)$

$= 10(107)$

$= 1070$

The first offer is better by \$70